FYJC - MATHEMATICS & STATISTICS

PAPER - II

MEASURES OF DISPERSION

SKEWNESS

MOMENTS

KURTOSIS

COEFFICIENT OF VARIATION

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

Q1. Find CV : 3 , 5 , 7 , 9 , 11

ans : σ = 2.829 , CV = 40.14%

Q2. Find CV : 20 22 19 23 26

ans : σ = 2.452 , CV = 11.15%

Q3. Find CV : 10 20 18 12 15

ans : σ = 3.689 , CV = 24.59%

Q4. Find CV : 35 , 40 , 20 , 45 , 30

ans : σ = 8.602 , CV = 25.3%

Q5. Find CV : 35 , 40 , 20 , 45 , 30

ans : σ = 6.722 , CV = 29.2 %

Q6. Calculate the coefficient of variation

15 , 16 , 18 , 18 , 19 , 20 , 20 , 21 , 21 , 22

ans : $\bar{x}$ = 19 ; σ = 2.145 ; CV =11.29 %

Q7.

Firm	A	B
No of employees	586	647
Mean Salary	52.5	47.5
S.D. of Salary	10	11

Which firm is homogenous with respect to payment of wages

Q1. Calculate the coefficient of variation

3 , 5 , 7 , 9 , 11

STEP 1 :

x	$x-\bar{x}$	$(x-\bar{x})^2$
3	−4	16
5	2	4
7	0	0
9	2	4
11	4	16
35		40

$$\bar{x} = \frac{\sum x}{n} = \frac{35}{5} = 7$$

STEP 2 :

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{40}{5}}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 8)$$

$$= \frac{1}{2}(0.9031)$$

$$= \frac{0.9031}{2}$$

$$\log \sigma = 0.4516$$

$$\sigma = AL(0.4516)$$

$$= 2.829$$

STEP 3 :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{2.829}{7} \times 100$$

$$= \frac{282.9}{7}$$

$$= 40.41\%$$

Q2 . Price of a certain commodity for the last 5 years in city A is given below

20 22 19 23 26

STEP 1 :

x	$x - \bar{x}$	$(x - \bar{x})^2$
20	– 2	4
22	0	0
19	– 3	9
23	1	1
26	4	16
110		30

$$\bar{x} = \frac{\Sigma x}{n} = \frac{110}{5} = 22$$

STEP 2 :

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{30}{5}} = \sqrt{6}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 6)$$

$$= \frac{1}{2}(0.7782)$$

$$= \frac{0.7782}{2}$$

$$\log \sigma = 0.3896$$

$$\sigma = AL(0.3896)$$

$$= 2.452$$

STEP 3 :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{2.452}{22} \times 100$$

$$= \frac{245.2}{22}$$

$$= 11.15\%$$

Q3 . Price of a certain commodity for the last 5 years in city A is given below

10 20 18 12 15

STEP 1 :

x	$x - \bar{x}$	$(x - \bar{x})^2$
10	– 5	25
20	5	25
18	3	9
12	–3	9
15	0	0
75		68

$$\bar{x} = \frac{\Sigma x}{n} = \frac{75}{5} = 15$$

STEP 2 :

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{68}{5}} = \sqrt{13.6}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 13.6)$$

$$= \frac{1}{2}(1.1335)$$

$$= \frac{1.1335}{2}$$

$$\log \sigma = 0.5668$$

$$\sigma = AL(0.5668)$$

$$= 3.689$$

STEP 3 :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{3.689}{15} \times 100$$

$$= \frac{368.9}{15}$$

$$= 24.59\%$$

Q4. Calculate the coefficient of variation

35 , 40 , 20 , 45 , 30

STEP 1 :

x	$x-\bar{x}$	$(x-\bar{x})^2$
35	1	1
40	6	36
20	−14	196
45	11	121
30	−4	16
170		370

$$\bar{x} = \frac{\sum x}{n} = \frac{170}{5} = 34$$

STEP 2 :

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{370}{5}} = \sqrt{74}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 74)$$

$$= \frac{1}{2}(1.8692)$$

$$= \frac{1.8692}{2}$$

$$\log \sigma = 0.9346$$

$$\sigma = AL(0.9346)$$

$$= 8.602$$

STEP 3 :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{8.602}{34} \times 100$$

$$= \frac{860.2}{34} = 25.3\%$$

Q5. Calculate the coefficient of variation

16 , 18 , 21 , 25 , 35

STEP 1 :

x	$x-\bar{x}$	$(x-\bar{x})^2$
16	− 7	49
18	− 5	25
21	− 2	4
25	2	4
35	12	144
115		226

$$\bar{x} = \frac{\sum x}{n} = \frac{115}{5} = 23$$

STEP 2 :

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{226}{5}} = \sqrt{45.2}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 45.2)$$

$$= \frac{1}{2}(1.6551)$$

$$= \frac{1.6551}{2}$$

$$\log \sigma = 0.8275$$

$$\sigma = AL(0.8275)$$

$$= 6.722$$

STEP 3 :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{6.722}{23} \times 100$$

$$= \frac{672.2}{23} = 29.2\ \%$$

Q6. Calculate the coefficient of variation

15 , 16 , 18 , 18 , 19 , 20 , 20 , 21 , 21 , 22

ans : $\bar{x} = 19$; $\sigma = 2.145$; CV $= 11.29\ \%$

Q7.

Firm	A	B
No of employees	586	647
Mean Salary	52.5	47.5
S.D. of Salary	10	11

Which firm is homogenous with respect to payment of wages

Firm A :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{10}{52.5} \times 100$$

$$= \frac{1000}{52.5} = 19.04\ \%$$

Firm B :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{11}{47.5} \times 100$$

$$= \frac{1100}{47.5} = 23.15\ \%$$

Since CV (A) < CV(B) , firm A is more homogenous with respect to payment of wages

CORRECTION OF MEAN & STANDARD DEVIATION

01.

the mean and standard deviation of 100 observations were found to be 6 and 2 respectively . If at the time of calculation , one observation was wrongly taken as 17 instead of 7 . Find the correct standard deviation

02.

the mean and standard deviation of set of 100 observations were worked out as 40 and 5 respectively by computer who by mistake took the value 50 in place of 40 for one of the observation . Find the corrected mean and standard deviation

03.

the mean and the variance of 12 items are 22 and 9 respectively . Later it was found that an item 32 was wrongly taken as 23 . Compute the correct mean and variance

04.

The mean and standard deviation of 9 items are 43 and 5 respectively . If an item of value 3 is added to the set find the mean and variance of the 10 items

05.

in a series of 5 observations , the value of mean and variance is 3 and 2 . If three observations are 1 , 3 & 5 find the remaining two

06.

in a series of 5 observations , the value of mean and variance is 4.4 and 8.24 . If three observations are 1 , 2 & 6 find the remaining two

01.

the mean and standard deviation of 100 observations were found to be 6 and 2 respectively . If at the time of calculation , one observation was wrongly taken as 17 instead of 7 . Find the correct standard deviation

$\bar{x} = 6$ & $\sigma = 2$, $n = 100$

incorrect x = 17 ,

correct x = 7

STEP 1 : CORRECTION OF MEAN

$$\bar{x} = \frac{\sum x}{n}$$

$$6 = \frac{\sum x}{100}$$

$\sum x$	=	600
– incorrect x	–	17
+correct x	+	7
$\sum x$ correct	=	590

$$\bar{x}_{correct} = \frac{\sum x}{n}$$

$$= \frac{590}{100}$$

$$= 5.9$$

STEP 2 : CORRECTION OF S.D.

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$\sum x^2 = n(\sigma^2 + \bar{x}^2)$$

$$= 100(2^2 + 6^2)$$

$$= 100(4 + 36)$$

$$= 4000$$

Now

$\sum x^2$	=	4000
– incorrect x^2	–	289
+ correct x^2	+	49
$\sum x^2$ correct	=	3760

$$\sigma_{correct} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

CORRECT MEAN

$$= \sqrt{\frac{3760}{100} - 5.9^2}$$

$$= \sqrt{37.60 - 34.81}$$

$$= \sqrt{2.79}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 2.79)$$

$$= \frac{1}{2}(0.4456)$$

$$= \frac{0.4456}{2}$$

$$\log \sigma = 0.2228$$

$$\sigma \text{ correct} = AL(0.2228)$$

$$= 1.670$$

02.

the mean and standard deviation of set of 100 observations were worked out as 40 and 5 respectively by computer who by mistake took the value 50 in place of 40 for one of the observation . Find the corrected mean and standard deviation

$\overline{x} = 40$, $\sigma = 5$, $n = 100$

incorrect x = 50 ,

correct x = 40

STEP 1 : CORRECTION OF MEAN

$$\overline{x} = \frac{\sum x}{n}$$

$$40 = \frac{\sum x}{100}$$

$\sum x$	=	4000
– incorrect x	–	50
+correct x	+	40
$\sum x$ correct	=	3990

$$\overline{x}_{correct} = \frac{\sum x}{n} = \frac{3990}{100} = 39.9$$

STEP 2 : CORRECTION OF S.D.

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \overline{x}^2$$

$$\sum x^2 = n(\sigma^2 + \overline{x}^2)$$

$$= 100(5^2 + 40^2)$$

$$= 100(25 + 1600)$$

$$= 162500$$

Now

$\sum x^2$	=	162500
– incorrect x^2	–	2500
+ correct x^2	+	1600
$\sum x^2$ correct	=	161600

$$\sigma_{correct} = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

$$= \sqrt{\frac{161600}{100} - 39.9^2}$$

$$= \sqrt{1616 - 1592.01}$$

$$= \sqrt{23.99}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 23.99)$$

$$= \frac{1.3801}{2}$$

$$= 0.69005$$

$$\log \sigma = 0.6901$$

σ correct= AL(0.6901) = 4.899

03.

the mean and the variance of 12 items are 22 and 9 respectively . Later it was found that an item 32 was wrongly taken as 23 . Compute the correct mean and variance.

$\overline{x} = 22$, $\sigma^2 = 9$, $n = 12$

incorrect x = 23 ,

correct x = 32

STEP 1 : CORRECTION OF MEAN

$$\overline{x} = \frac{\sum x}{n}$$

$$22 = \frac{\sum x}{12}$$

$\sum x$	=	264
– incorrect x	–	23
+correct x	+	32
$\sum x$ correct	=	273

$$\overline{x}_{correct} = \frac{\sum x}{n}$$

$$= \frac{273}{12}$$

$$= 22.75$$

STEP 2 : CORRECTION OF S.D.

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \overline{x}^2$$

$$\sum x^2 = n(\sigma^2 + \overline{x}^2)$$

$$= 12(9 + 22^2)$$

$$= 12(9 + 484)$$

$$= 12(493)$$

$$= 5916$$

Now

$\sum x^2$	=	5916
– incorrect x^2	–	529
+ correct x^2	+	1024
$\sum x^2$ correct	=	6411

$$\sigma_{correct} = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

CORRECT MEAN

$$\sigma^2_{correct} = \frac{\sum x^2}{n} - \overline{x}^2$$

$$\text{variance} = \frac{6411}{12} - 22.75^2$$

$$= 534.25 - 517.5625$$

$$= 16.6875$$

04.

The mean and standard deviation of 9 items are 43 and 5 respectively . If an item of value 3 is added to the set find the mean and variance of the 10 items .

$\overline{x} = 43$, $\sigma = 5$, $n = 9$

new x added = 3 ,

STEP 1 : NEW MEAN

$$\overline{x} = \frac{\sum x}{n}$$

$$43 = \frac{\sum x}{9}$$

$\sum x$	=	387
+new x	+	3
$\sum x$ new	=	390

$$\overline{x}_{new} = \frac{\sum x}{n}$$

$$= \frac{390}{10}$$

$$= 39$$

STEP 2 : CORRECTION OF S.D.

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \overline{x}^2$$

$$\sum x^2 = n(\sigma^2 + \overline{x}^2)$$

$$= 9(5^2 + 43^2)$$

$$= 9(25 + 1849)$$

$$= 9(1874)$$

$$= 16866$$

Now

$\sum x^2$	=	16866
+ new x^2	+	9
$\sum x^2$ new	=	16875

$$\sigma_{new} = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

NEW MEAN

$$\sigma^2_{new} = \frac{\sum x^2}{n} - \overline{x}^2$$

$$\text{variance} = \frac{16875}{10} - 39^2$$

$$= 1687.5 - 1521$$

$$= 166.5$$

05.

in a series of 5 observations , the value of mean and variance is 3 and 2 . If three observations are 1 , 3 & 5 find the remaining two

let the other 2 observations be a & b

$$\bar{x} = \frac{\sum x}{n}$$

$$3 = \frac{1 + 3 + 5 + a + b}{5}$$

$$15 = 9 + a + b$$

$$a + b = 6$$

$$\therefore b = 6 - a \quad \ldots\ldots (1)$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$2 = \frac{1 + 9 + 25 + a^2 + b^2}{5} - 3^2$$

$$2 = \frac{35 + a^2 + b^2}{5} - 9$$

$$11 = \frac{35 + a^2 + b^2}{5}$$

$$55 = 35 + a^2 + b^2$$

$$a^2 + b^2 = 20$$

$$a^2 + (6 - a)^2 = 20 \quad \text{from (1)}$$

$$a^2 + 36 - 12a + a^2 = 20$$

$$2a^2 - 12a + 16 = 0$$

$$a^2 - 6a + 8 = 0$$

$a = 4$ $\qquad$ $a = 2$

$b = 6 - a$ $\qquad$ $b = 6 - a$

$b = 2$ $\qquad$ $b = 4$

$\therefore$ the other two observations are 2 & 4

06.

in a series of 5 observations , the value of mean and variance is 4.4 and 8.24 . If three observations are 1 , 2 & 6 find the remaining two

let the other 2 observations be a & b

$$\bar{x} = \frac{\sum x}{n}$$

$$4.4 = \frac{1 + 2 + 6 + a + b}{5}$$

$$22 = 9 + a + b$$

$$a + b = 13$$

$$\therefore b = 13 - a \quad \ldots\ldots (1)$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$8.24 = \frac{1 + 4 + 36 + a^2 + b^2}{5} - 4.4^2$$

$$8.24 = \frac{41 + a^2 + b^2}{5} - 19.36$$

$$27.6 = \frac{41 + a^2 + b^2}{5}$$

$$138 = 41 + a^2 + b^2$$

$$a^2 + b^2 = 97$$

$$a^2 + (13 - a)^2 = 97 \quad \ldots\ldots \text{from (1)}$$

$$a^2 + 169 - 26a + a^2 = 97$$

$$2a^2 - 26a + 72 = 0$$

$$a^2 - 13a + 36 = 0$$

$a = 9$ $\qquad$ $a = 4$

$b = 13 - a$ $\qquad$ $b = 13 - a$

$b = 4$ $\qquad$ $b = 9$

$\therefore$ the other two observations are 4 & 9

SKEWNESS

In symmetrical distribution the mean and mode coincide

But in skewed distribution , they don't and hence the difference between them measures the amount of skewness

Measure of skewness = Mean – Mode

Coefficient of skewness ;

$$SKp = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

However if Mode is ill – defined , we make use of empirical relationship between Mean – Median – Mode

Mean – Mode = 3(Mean – Median)

In that case $$SKp = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

KARL PEARSON'S COEFF. OF SKEWNESS

Q1. Find Karl Pearson's coefficient of skewness : 6 , 5 , 7 , 0 , 2
ans : SkP = −1.15

Q2. Find Karl Pearson's coefficient of skewness : 9 , 6 , 5 , 11 , 13 , 12 , 14
ans : SkP = 3.21

Q3. Find Karl Pearson's coefficient of skewness :
n = 10 ; $\Sigma x = 450$; $\Sigma x^2 = 24250$; Mode = 43
ans : SkP = 0.1

Q4. For a moderately skewed distribution Mean = 200 ; median = 198.4 , SD = 16 Find mode and the Pearsons coefficient of skewness (SKp) **ans :** SkP = 0.3

Q5. the mean & variance of a distribution are 50 and 400 respectively . Find the mode and the median if SKp = –0.4
ans : mode = 58 & median = 52.67

Q6. SKp = –0.4 ; SD = 20 ; CV = 40% . Find mean ; median & mode
ans : 50 , 52.67 , 58

Q7. for moderately skewed distribution mean = 40 ; karlpearsons coefficient of skewness is 0.1 & coeff. Of variation is 20% . Find mode

Q8. Mean = 200 , coefficient of variation is 8% and Karl Persons's coefficient of skewness (SKp) = 0.3 . Find mode & median
ans : mode = 195.2 & median = 198.4

Q9. SKp = 0.06 ; mean = 150 ; var. = 2500 Find : median ; mode & CV
ans : 149 , 147 , 33.33%

SOLUTION SET

Q1. Find Karl Pearson's coefficient of skewness : 6 , 5 , 7 , 0 , 2

STEP 1 : MEAN

$$\overline{x} = \frac{\sum x}{N} = \frac{20}{5} = 4$$

STEP 2 : MEDIAN

Obs no. :	1	2	3	4	5
Value :	0	2	5	6	7

Median = value of $\frac{N+1}{2}$ observation

= value of 3rd observation

= 5

STEP 3 : STANDARD DEVIATION

x	$x - \overline{x}$	$(x - \overline{x})^2$
0	−4	16
2	−2	4
5	1	1
6	2	4
7	3	9
	0	34

$$\sigma = \sqrt{\frac{\sum(x - \overline{x})^2}{n}}$$

$$= \sqrt{\frac{34}{5}}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 6.8)$$

$$= \frac{1}{2}(0.8325)$$

$$= \frac{0.8325}{2}$$

$\log \sigma = 0.4163$

$\sigma = AL(0.4163)$

$= 2.608$

$= 2.61$

STEP 4 : KARL PERASON COEFF. OF SKEWNESS

$$Sk_P = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

$$= \frac{3(4-5)}{2.61}$$

$$= -\frac{3}{2.61}$$

$$= -\frac{300}{261} = -1.15$$

Q2. Find Karl Pearson's coefficient of skewness : 9 , 6 , 5 , 11 , 13 , 12 , 14

STEP 1 : MEAN

$$\overline{x} = \frac{\sum x}{N} = \frac{70}{7} = 10$$

STEP 2 : MEDIAN

Obs no.:	1	2	3	4	5	6	7
Value :	5	6	9	11	12	13	14

Median = value of $\frac{N+1}{2}$ observation

= value of 4TH observation

= 11

STEP 3 : STANDARD DEVIATION

x	$x - \overline{x}$	$(x - \overline{x})^2$
5	−5	25
6	−4	16
9	−1	1
11	1	1
12	2	4
13	3	9
14	4	16
	0	72

$$\sigma = \sqrt{\frac{\sum(x - \overline{x})^2}{n}}$$

$$= \sqrt{\frac{72}{7}} = \sqrt{10.29}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 10.29)$$

$$= \frac{1}{2}(1.0123)$$

$$= \frac{1.0123}{2}$$

$$\log \sigma = 0.5062$$

$$\sigma = AL(0.5062)$$

$$= 3.207$$

$$= 3.21$$

STEP 4 : KARL PERASON COEFF. OF SKEWNESS

$$Sk_P = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

$$= \frac{3(10 - 11)}{3.21}$$

$$= -\frac{3}{3.21}$$

$$= -\frac{300}{321}$$

$$= -0.93$$

Q3. Find Karl Pearson's coefficient of skewness :

$n = 10$; $\Sigma x = 450$; $\Sigma x^2 = 24250$;

Mode = 43

STEP 1 : MEAN

$$\bar{x} = \frac{\Sigma x}{N} = \frac{450}{10} = 45$$

STEP 2 : STANDARD DEVIATION

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$\sigma = \sqrt{\frac{24250}{10} - 45^2}$$

$$= \sqrt{2425 - 2025}$$

$$= \sqrt{400}$$

$$= 20$$

STEP 3 : KARL PERASON COEFF. OF SKEWNESS

$$Sk_P = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$= \frac{45 - 43}{20}$$

$$= \frac{2}{20}$$

$$= 0.1$$

Q4. For a moderately skewed distribution Mean = 200 ; median = 198.4 , SD = 16 Find mode and the Pearsons coefficient of skewness (SKp)

STEP 1 : MODE

Mean – mode = 3(mean - median)

200 – mode = 3(200 – 198.4)

200 – mode = 3(1.6)

200 – mode = 4.8

mode = 200 – 4.8

= 195.2

STEP 2 :KARL PERASON COEFF. OF SKEWNESS

$$Sk_P = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

$$= \frac{3(200 - 198.4)}{16}$$

$$= \frac{3(\cancel{16})}{\cancel{16}\ 10} = \frac{3}{10} = 0.3$$

Q5. the mean & variance of a distribution are 50 and 400 respectively . Find the mode and the median if SKp = –0.4

Mean = 50 , σ^2 = 400 , SKp = –0.4

STEP 1 : MODE

$$Sk_P = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$-0.4 = \frac{50 - \text{Mode}}{20}$$

$$-8 = 50 - \text{Mode}$$

$$\text{Mode} = 50 + 8$$

$$= 58$$

STEP 2 : MEDIAN

$$\text{Mean} - \text{mode} = 3(\text{mean} - \text{median})$$

$$50 - 58 = 3(50 - \text{median})$$

$$-\frac{8}{3} = 50 - \text{median}$$

$$-2.67 = 50 - \text{median}$$

$$\text{median} = 52,67$$

Q6. SKp = –0.4 ; SD = 20 ; CV = 40% . Find mean ; median & mode

STEP 1 : MEAN

$$CV = \frac{\sigma}{\overline{x}} \times 100$$

$$40 = \frac{20}{\overline{x}} \times 100$$

$$\overline{x} = \frac{20 \times 100}{40} = 50$$

STEP 2 : MODE

$$Sk_P = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$-0.4 = \frac{50 - \text{Mode}}{20}$$

$$-8 = 50 - \text{Mode}$$

$$\text{Mode} = 50 + 8$$

$$\text{Mode} = 58$$

STEP 3 : MEDIAN

$$\text{Mean} - \text{mode} = 3(\text{mean} - \text{median})$$

$$50 - 58 = 3(50 - \text{median})$$

$$-\frac{8}{3} = 50 - \text{median}$$

$$-2.67 = 50 - \text{median}$$

$$\text{median} = 52,67$$

Q7. for moderately skewed distribution mean = 40 ; karlpearsons coefficient of skewness is 0.1 & coeff. of variation is 20% . Find mode

STEP 1 : SD

$$CV = \frac{\sigma}{\overline{x}} \times 100$$

$$20 = \frac{\sigma}{40} \times 100$$

$$\sigma = 8$$

STEP 2 : MODE

$$Sk_P = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$0.1 = \frac{40 - \text{Mode}}{8}$$

$$0.8 = 40 - \text{Mode}$$

$$\text{Mode} = 40 - 0.8$$

$$\text{Mode} = 39.2$$

STEP 3 : MEDIAN

$$\text{Mean} - \text{mode} = 3(\text{mean} - \text{median})$$

$$40 - 39.2 = 3(40 - \text{median})$$

$$0.8 = 3(40 - \text{median})$$

$$0.27 = 40 - \text{median}$$

$$\text{median} = 40 - 0.27 = 39.73$$

Q8. Mean = 200 , coefficient of variation is 8% and Karl Persons's coefficient of skewness (SKp) = 0.3 . Find mode & median

STEP 1 : SD

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$8 = \frac{\sigma}{200} \times 100$$

$$\sigma = 16$$

STEP 2 : MODE

$$Skp = \frac{Mean - Mode}{\sigma}$$

$$0.3 = \frac{200 - Mode}{16}$$

$$4.8 = 200 - Mode$$

$$Mode = 200 - 4.8$$

$$Mode = 195.2$$

STEP 3 : MEDIAN

Mean – mode = 3(mean - median)

200 – 195.2 = 3(200 – median)

4.8 = 3(200 – median)

1.6 = 200 – median

median = 200 – 1.6

median = 198.4

Q9.

SKp = 0.06 ; mean = 150 ; variance = 2500
Find : median ; mode & CV

STEP 1 : MODE

$$Skp = \frac{Mean - Mode}{\sigma} \qquad (\text{given } \sigma^2 = 2500)$$

$$0.06 = \frac{150 - Mode}{50}$$

$$\frac{6}{100} = \frac{150 - Mode}{50}$$

$$3 = 150 - Mode$$

$$Mode = 150 - 3$$

$$Mode = 147$$

STEP 2 : MEDIAN

Mean – mode = 3(mean - median)

150 – 147 = 3(150 – median)

3 = 3(150 – median)

1 = 150 – median

median = 150 – 1

median = 149

STEP 3 : CV

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{50}{150} \times 100$$

$$= 33.33\%$$

SKEWNESS

In symmetrical distribution the quartiles are equidistant from the median

$$Q_2 - Q_1 = Q_3 - Q_2$$

But in skewed distribution ,
the quartiles will not be equidistant from the median and hence the difference between them will measure the amount of skewness

BOWLEY'S COEFFICIENT OF SKEWNESS

Bowley's absolute measure of skewness

$$= (Q_3 - Q_2) - (Q_2 - Q_1)$$
$$= Q_3 + Q_1 - 2Q_2$$

Bowley's coefficient of skewness

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

Q SET

01.
The lower and upper quartiles are 15 and 21 respectively and its median is 17 . Find Bowley's coefficient of skewness .
ans : $SK_B = 0.33$

02.
For a frequency distribution ;
$Q_3 - Q_2 = 40$ & $Q_2 - Q_1 = 60$. Find SK_B
ans : $SK_B = -0.2$

03.
For a frequency distribution ;
$Q_3 - Q_2 = 100$ & $Q_2 - Q_1 = 120$. Find SK_B
ans : $SK_B = 0.09$

04. Find Bowley's coefficient of skewness
a) 168 , 164 , 172 , 169 , 178 , 173 , 173
b) 29 , 12 , 24 , 19 , 26 , 36 , 35 , 21 , 33
c) 160 , 158 , 153 , 161 , 152 , 157 , 162 , 159 , 156 , 165
ans : SK_B = a) – 0.6 b) 0.14 c) – 0 .083

05.
for a frequency distribution , Bowley's coefficient of skewness is –0.8 . If $Q_1 = 44.1$ and $Q_3 = 56.6$, find the median of the distribution
ans : $M = 55.35$

06.
Bowley's coefficient of skewness is 0.6 .
The sum of upper and lower quartiles is 100 and the median is 38 . Find the upper and lower quartiles
ans : $Q_1 = 30$; $Q_3 = 70$

07.
if median , first quartile and coefficient of quartile deviation of distribution are 18.5 , 14.5 and 0.275 respt. Calculate Bowley's coefficient of skewness
ans : $SK_B = 0.27$

01.

The lower and upper quartiles are 15 and 21 respectively and its median is 17 . Find Bowley's coefficient of skewness .

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

$$= \frac{21 + 15 - (2(17)}{21 - 15}$$

$$= \frac{36 - 34}{6}$$

$$= \frac{2}{6}$$

$$= 0.33$$

02.

For a frequency distribution ;

$Q_3 - Q_2 = 40$ & $Q_2 - Q_1 = 60$. Find SK_B

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{40 - 60}{40 + 60}$$

$$= \frac{-20}{100}$$

$$= -0.2$$

03.

For a frequency distribution ;

$Q_3 - Q_2 = 100$ & $Q_2 - Q_1 = 120$. Find SK_B

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{100 - 120}{100 + 120}$$

$$= \frac{-20}{220}$$

$$= \frac{-1}{11}$$

$$= 0.09$$

04. Find Bowley's coefficient of skewness

a) 168 , 164 , 172 , 169 , 178 , 173 , 173

1	2	3	4	5	6	7
164	168	169	172	178	173	173

STEP 1 :

$$q_1 = \frac{N + 1}{4} = \frac{8}{4} = 2$$

Q_1 = value of the 2nd observation

= 168

$$q_2 = \frac{N + 1}{2} = \frac{8}{2} = 4$$

Q_2 = value of the 4th observation

= 172

$$q_3 = \frac{3(N + 1)}{4} = 3(2) = 6$$

Q_3 = value of the 6th observation

= 173

STEP 2 :

$Q_3 - Q_2 = 173 - 172 = 1$

$Q_2 - Q_1 = 172 - 168 = 4$

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{1 - 4}{1 + 4}$$

$$= \frac{-3}{5}$$

$$= -0.6$$

b) 29 , 12 , 24 , 19 , 26 , 36 , 35 , 21 , 33

1	2	3	4	5	6	7	8	9
12	19	21	24	26	29	33	35	36

STEP 1 :

$q_1 = \frac{N+1}{4} = \frac{10}{4} = 2.5$

Q_1 = value of the 2.5^{th} observation

$= 19 + 0.5(21 - 19)$

$= 19 + 0.5(2)$

$= 19 + 1 = 20$

$q_2 = \frac{N+1}{2} = \frac{10}{2} = 5$

Q_2 = value of the 5^{th} observation

$= 26$

$q_3 = \frac{3(N+1)}{4} = 3(2.5) = 7.5$

Q_3 = value of the 7.5^{th} observation

$= 33 + 0.5(35 - 33)$

$= 33 + 0.5(2)$

$= 33 + 1$

$= 34$

STEP 2 :

$Q_3 - Q_2 = 34 - 26 = 8$

$Q_2 - Q_1 = 26 - 20 = 6$

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{8 - 6}{8 + 6}$$

$$= \frac{2}{14}$$

$$= \frac{1}{7}$$

$$= 0.14$$

c) 160 , 158 , 153 , 161 , 152 , 157 , 162 , 159 , 156 , 165

1	2	3	4	5	6	7
152	153	156	157	158	159	160

8	9	10
161	162	165

STEP 1 :

$q_1 = \frac{N+1}{4} = \frac{11}{4} = 2.75$

Q_1 = value of the 2.75^{th} observation

$= 153 + 0.75(56 - 53)$

$= 153 + 0.75(3)$

$= 153 + 2.25 = 155.25$

$q_2 = \frac{N+1}{2} = \frac{11}{2} = 5.5$

Q_2 = value of the 5.5^{th} observation

$= 158 + 0.5(159 - 158)$

$= 158 + 0.5(1)$

$= 158 + 0.5 = 158.5$

$q_3 = \frac{3(N+1)}{4} = 3(2.75) = 8.25$

Q_3 = value of the 8.25^{th} observation

$= 161 + 0.25(162 - 161)$

$= 161 + 0.25(1)$

$= 161 + 0.25$

$= 161.25$

STEP 2 :

$Q_3 - Q_2 = 161.25 - 158.5 = 2.75$

$Q_2 - Q_1 = 158.5 - 155.25 = 3.25$

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{2.75 - 3.25}{2.75 + 3.25}$$

$$= \frac{-0.5}{6}$$

$$= -0.083$$

05.

for a frequency distribution , Bowley's coefficient of skewness is –0.8 . If $Q_1 = 44.1$ and $Q_3 = 56.6$, find the median of the distribution

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$SK_B = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

$$-0.8 = \frac{56.6 + 44.1 - 2M}{56.6 - 44.1}$$

$$-0.8 = \frac{100.7 - 2M}{12.5}$$

$$-10 = 100.7 - 2M$$

$$2M = 110.7$$

$$M = 55.35$$

06.

Bowley's coefficient of skewness is 0.6 .

The sum of upper and lower quartiles is 100 and the median is 38 . Find the upper and lower quartiles

$Q_3 + Q_1 = 100$; $M = 38$; $SK_B = 0.6$

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$SK_B = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

$$0.6 = \frac{100 - 2(38)}{Q_3 - Q_1}$$

$$Q_3 - Q_1 = \frac{100 - 76}{0.6}$$

$$Q_3 - Q_1 = \frac{24}{0.6} = \frac{240}{6} = 40$$

$$\begin{aligned} \text{Now} \quad Q_3 + Q_1 &= 100 \\ Q_3 - Q_1 &= 40 \\ \hline 2\,Q_3 &= 140 \end{aligned}$$

$$Q_3 = 70$$

Subs in

$$Q_3 + Q_1 = 100$$

$$70 + Q_1 = 100$$

$$Q_1 = 30$$

07.

if median , first quartile and coefficient of quartile deviation of distribution are 18.5 , 14.5 and 0.275 respt. Calculate Bowley's coefficient of skewness

$Q_1 = 14.5$, $M = 18.5$, coeff of Q.D. = 0.275

coeff of Q.D. = 0.275

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{\cancel{275}^{\,11}}{\cancel{1000}_{\,40}}$$

$$40Q_3 - 40Q_1 = 11Q_3 + 11Q_1$$

$$29Q_3 = 51Q_1$$

$$Q_3 = \frac{51Q_1}{29}$$

$$Q_3 = \frac{51(14.5)}{29}$$

$$= \frac{51(\cancel{145})^{5}}{\cancel{29} \times 10}$$

$$Q_3 = 25.5$$

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$SK_B = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

$$= \frac{25.5 + 14.5 - 2(18.5)}{25.5 - 14.5}$$

$$= \frac{40 - 37}{11}$$

$$= \frac{3}{11}$$

$$= 0.27$$

MOMENTS

1. – Central Moments (Moments about mean)

2. – Raw Moments , Moments about arbitrary value A

3. – Convert raw moments or moments about A to central moments

MOMENTS ABOUT ARBITRARY VALUE

$$\mu_{1(A)} = \frac{\Sigma (x - A)}{n}$$

$$\mu_{2(A)} = \frac{\Sigma (x - A)^2}{n}$$

$$\mu_{3(A)} = \frac{\Sigma (x - A)^3}{n}$$

$$\mu_{4(A)} = \frac{\Sigma (x - A)^4}{n}$$

CENTRAL MOMENTS (MOMENTS ABOUT MEAN)

$$\mu_1 = \frac{\Sigma (x - \bar{x})}{n}$$

$$\mu_2 = \frac{\Sigma (x - \bar{x})^2}{n}$$

$$\mu_3 = \frac{\Sigma (x - \bar{x})^3}{n}$$

$$\mu_4 = \frac{\Sigma (x - \bar{x})^4}{n}$$

RAW MOMENTS (MOMENTS ABOUT ORIGIN)

$\mu_1' = \frac{\Sigma x}{n}$; $\mu_2' = \frac{\Sigma x^2}{n}$; $\mu_3' = \frac{\Sigma x^3}{n}$; $\mu_4' = \frac{\Sigma x^4}{n}$

Q1 : MOMENTS ABOUT MEAN (CENTRAL MOMENTS)

01. Find moments about mean for : 5 , 4 , 7 , 6 , 3

x	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^3$	$(x - \bar{x})^4$
3	−2	4	−8	16
4	−1	1	−1	1
5	0	0	0	0
6	1	1	1	1
7	2	4	8	16
$\bar{x} = 5$	0	10	0	34
	$\Sigma(x - \bar{x})$	$\Sigma(x - \bar{x})^2$	$\Sigma(x - \bar{x})^3$	$\Sigma(x - \bar{x})^4$

CENTRAL MOMENTS :

$$\mu_1 = \frac{\Sigma (x - \bar{x})}{n} = 0$$

$$\mu_2 = \frac{\Sigma (x - \bar{x})^2}{n} = \frac{10}{5} = 2$$

$$\mu_3 = \frac{\Sigma (x - \bar{x})^3}{n} = 0$$

$$\mu_4 = \frac{\Sigma (x - \bar{x})^4}{n} = \frac{34}{5} = 6.8$$

02. Find moments about mean for : 2 , 4 , 5 , 8 , 11

x	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
2	−4	16	−64	256
4	−2	4	$\cancel{-8}$	16
5	−1	1	−1	1
8	2	4	$\cancel{8}$	16
11	5	25	125	625
$\bar{x} = 6$	0	50	60	914
	$\Sigma(x-\bar{x})$	$\Sigma(x-\bar{x})^2$	$\Sigma(x-\bar{x})^3$	$\Sigma(x-\bar{x})^4$

CENTRAL MOMENTS :

$$\mu_1 = \frac{\Sigma(x-\bar{x})}{n} = 0$$

$$\mu_2 = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{50}{5} = 10$$

$$\mu_3 = \frac{\Sigma(x-\bar{x})^3}{n} = \frac{60}{5} = 12$$

$$\mu_4 = \frac{\Sigma(x-\bar{x})^4}{n} = \frac{914}{5} = 182.8$$

03. Find moments about mean for : 16 , 19 , 22 , 23 , 25

x	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
16	−5	25	−125	625
19	−2	4	$\cancel{-8}$	16
22	1	1	1	1
23	2	4	$\cancel{8}$	16
25	4	16	64	256
$\bar{x} = 21$	0	50	−60	914
	$\Sigma(x-\bar{x})$	$\Sigma(x-\bar{x})^2$	$\Sigma(x-\bar{x})^3$	$\Sigma(x-\bar{x})^4$

CENTRAL MOMENTS :

$$\mu_1 = \frac{\Sigma(x-\bar{x})}{n} = 0$$

$$\mu_2 = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{50}{5} = 10$$

$$\mu_3 = \frac{\Sigma(x-\bar{x})^3}{n} = \frac{-60}{5} = -12$$

$$\mu_4 = \frac{\Sigma(x-\bar{x})^4}{n} = \frac{914}{5} = 182.8$$

04. Find moments about mean for : −3 , −2 , 0 , 4 , 6

x	x − 1	$(x-1)^2$	$(x-1)^3$	$(x-1)^4$
−3	−4	16	−64	256
−2	−3	9	$\cancel{-9}$	81
0	−1	1	−1	1
4	3	9	$\cancel{9}$	81
6	5	25	125	625
$\bar{x} = 1$	0	60	60	1044
	$\Sigma(x-1)$	$\Sigma(x-1)^2$	$\Sigma(x-1)^3$	$\Sigma(x-1)^4$

CENTRAL MOMENTS :

$$\mu_1 = \frac{\Sigma(x-\bar{x})}{n} = 0$$

$$\mu_2 = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{60}{5} = 12$$

$$\mu_3 = \frac{\Sigma(x-\bar{x})^3}{n} = \frac{60}{5} = 12$$

$$\mu_4 = \frac{\Sigma(x-\bar{x})^4}{n} = \frac{1044}{5} = 208.8$$

Q2 : MOMENTS ABOUT ARBITRARY VALUE 'A'

01. find moments about A = 5 : 5 , 8 , 7 , 4 , 6

x	x − A	$(x-A)^2$	$(x-A)^3$	$(x-A)^4$
4	−1	1	−1	1
5	0	0	0	0
6	1	1	1	1
7	2	4	8	16
8	3	9	27	81
A = 5	5	15	35	99
	$\Sigma(x-A)$	$\Sigma(x-A)^2$	$\Sigma(x-A)^3$	$\Sigma(x-A)^4$

MOMENTS ABOUT '5'

$$\mu_{1(A)} = \frac{\Sigma(x-A)}{n} = \frac{5}{5} = 1$$

$$\mu_{2(A)} = \frac{\Sigma(x-A)^2}{n} = \frac{15}{5} = 3$$

$$\mu_{3(A)} = \frac{\Sigma(x-A)^3}{n} = \frac{35}{5} = 7$$

$$\mu_{4(A)} = \frac{\Sigma(x-A)^4}{n} = \frac{99}{5} = 19.8$$

02. find moments about A = 5 : 7 , 8 , 6 , 5

x	x − A	$(x-A)^2$	$(x-A)^3$	$(x-A)^4$
5	0	0	0	0
6	1	1	1	1
7	2	4	8	16
8	3	9	27	81
A = 5	6	14	36	98
	$\Sigma(x-A)$	$\Sigma(x-A)^2$	$\Sigma(x-A)^3$	$\Sigma(x-A)^4$

MOMENTS ABOUT '5'

$$\mu_{1(A)} = \frac{\Sigma(x-A)}{n} = \frac{6}{4} = 1.5$$

$$\mu_{2(A)} = \frac{\Sigma(x-A)^2}{n} = \frac{14}{4} = 3.5$$

$$\mu_{3(A)} = \frac{\Sigma(x-A)^3}{n} = \frac{36}{4} = 9$$

$$\mu_{4(A)} = \frac{\Sigma(x-A)^4}{n} = \frac{98}{4} = 24.5$$

03. find moments about 20 : 23 , 20 , 19 , 22 , 19

x	x − A	$(x-A)^2$	$(x-A)^3$	$(x-A)^4$
19	−1	1	−1	1
19	−1	1	−1	1
20	0	0	0	0
22	2	4	8	16
23	3	9	27	81
A = 5	3	15	33	99
	$\Sigma(x-A)$	$\Sigma(x-A)^2$	$\Sigma(x-A)^3$	$\Sigma(x-A)^4$

MOMENTS ABOUT '20'

$$\mu_{1(A)} = \frac{\Sigma(x-A)}{n} = \frac{3}{5} = 0.6$$

$$\mu_{2(A)} = \frac{\Sigma(x-A)^2}{n} = \frac{15}{5} = 3$$

$$\mu_{3(A)} = \frac{\Sigma(x-A)^3}{n} = \frac{33}{5} = 6.6$$

$$\mu_{4(A)} = \frac{\Sigma(x-A)^4}{n} = \frac{99}{5} = 19.8$$

04. find raw moments for : −3 , −2 , 1 , 3 , 4

x	x^2	x^3	x^4
−3	9	−27	81
−2	4	− 8	16
1	1	1	1
3	9	27	81
4	16	64	256
3	39	57	435
Σx	Σx^2	Σx^3	Σx^4

RAW MOMENTS :

$$\mu'_1 = \frac{\Sigma x}{n} = \frac{3}{5} = 0.6$$

$$\mu'_2 = \frac{\Sigma x^2}{n} = \frac{39}{5} = 7.8$$

$$\mu'_3 = \frac{\Sigma x^3}{n} = \frac{57}{5} = 11.4$$

$$\mu'_4 = \frac{\Sigma x^4}{n} = \frac{435}{5} = 87$$

CONVERSION OF MOMENTS ABOUT ARBITRARY VALUE / RAW MOMENTS TO CENTRAL MOMENTS

$$\mu_1 = 0$$

$$\mu_2 = \mu_2(a) - \mu_1(a)^2$$

$$\mu_3 = \mu_3(a) - 3\,\mu_1(a)\,.\,\mu_2(a) + 2\,\mu_1(a)^3$$

$$\mu_4 = \mu_4(a) - 4\,\mu_1(a)\,.\,\mu_3(a) + 6\,\mu_2(a)\,.\mu_1(a)^2 - 3\mu_1(a)^4$$

NOTE :

$$\mu_{1(a)} = \bar{x} - A$$

$$\text{VARIANCE} = \sigma^2 = \frac{\Sigma(x - x)^2}{n} = \mu_2$$

01. moments about 5 : 2 , 10 , 50 , 230

Find central moments

SOLUTION :

A = 5 ; $\mu_1(a) = 2$; $\mu_2(a) = 10$; $\mu_3(a) = 50$; $\mu_4(a) = 230$

$\mu_1 = 0$

$$\begin{aligned}\mu_2 &= \mu_2(a) - \mu_1(a)^2\\ &= 10 - (2)^2\\ &= 10 - 4\\ &= 6\end{aligned}$$

$$\begin{aligned}\mu_3 &= \mu_3(a) - 3\mu_1(a)\,.\,\mu_2(a) + 2\mu_1(a)^3\\ &= 50 - 3(2)(10) + 2(2)^3\\ &= 50 - 60 + 16\\ &= 6\end{aligned}$$

$$\begin{aligned}\mu_4 &= \mu_4(a) - 4\mu_1(a)\,.\,\mu_3(a) + 6\mu_2(a)\cdot\mu_1(a)^2 - 3\mu_1(a)^4\\ &= 230 - 4(2)(50) + 6(10)(2)^2 - 3(2)^4\\ &= 230 - 400 + 240 - 48\\ &= 470 - 448\\ &= 22\end{aligned}$$

$$\begin{aligned}\mu_1(a) &= \overline{x} - A\\ 2 &= \overline{x} - 5\\ \overline{x} &= 7\end{aligned}$$

02. first four raw moments : 2 , 20 , 40 , 50

Find central moments

SOLUTION :

A = 0 ; $\mu_1(a) = 2$; $\mu_2(a) = 20$; $\mu_3(a) = 40$; $\mu_4(a) = 50$

RAW

TO AVOID MANY NOTATIONS IN THIS SUM WE HAVE CALLED $\mu_1(a)$ INSTEAD OF μ_1' AND SO ON

HOWEVER WE HAVE MENTIONED A=0 SO THAT READER UNDERSTANDS THAT THEY ARE RAW MOMENTS

$\mu_1 = 0$

$$\begin{aligned}\mu_2 &= \mu_2(a) - \mu_1(a)^2\\ &= 20 - (2)^2\\ &= 20 - 4\\ &= 16\end{aligned}$$

$$\begin{aligned}\mu_3 &= \mu_3(a) - 3\mu_1(a)\,.\,\mu_2(a) + 2\mu_1(a)^3\\ &= 50 - 3(2)(20) + 2(2)^3\\ &= 40 - 120 + 16\\ &= -64\end{aligned}$$

$$\begin{aligned}\mu_4 &= \mu_4(a) - 4\mu_1(a)\,.\,\mu_3(a) + 6\mu_2(a)\cdot\mu_1(a)^2 - 3\mu_1(a)^4\\ &= 50 - 4(2)(40) + 6(20)(2)^2 - 3(2)^4\\ &= 50 - 320 + 480 - 48\\ &= 530 - 368\\ &= 162\end{aligned}$$

$$\begin{aligned}\mu_1(a) &= \overline{x} - A\\ 2 &= \overline{x} - 0\\ \overline{x} &= 2\end{aligned}$$

03. first four raw moments : 2 , 7 , 20 , 76

Find central moments

SOLUTION :

A = 0 ; $\mu_1(a)$ = 2 ; $\mu_2(a)$ = 7 ; $\mu_3(a)$ = 20 ; $\mu_4(a)$ = 76

RAW MOMENTS

TO AVOID MANY NOTATIONS IN THIS SUM WE HAVE CALLED $\mu_1(a)$ INSTEAD OF μ_1' AND SO ON
HOWEVER WE HAVE MENTIONED A=0 SO THAT READER UNDERSTANDS THAT THEY ARE RAW MOMENTS

$\mu_1 = 0$

$$\begin{aligned}\mu_2 &= \mu_2(a) - \mu_1(a)^2\\ &= 7 - (2)^2\\ &= 7 - 4\\ &= 3\end{aligned}$$

$$\begin{aligned}\mu_3 &= \mu_3(a) - 3\mu_1(a)\,.\,\mu_2(a) + 2\mu_1(a)^3\\ &= 20 - 3(2)(7) + 2(2)^3\\ &= 20 - 42 + 16\\ &= -6\end{aligned}$$

$$\begin{aligned}\mu_4 &= \mu_4(a) - 4\mu_1(a)\,.\,\mu_3(a) + 6\mu_2(a)\cdot\mu_1(a)^2 - 3\mu_1(a)^4\\ &= 76 - 4(2)(20) + 6(7)(2)^2 - 3(2)^4\\ &= 76 - 160 + 168 - 48\\ &= 244 - 208\\ &= 36\end{aligned}$$

$$\begin{aligned}\mu_1(a) &= \overline{x} - A\\ 2 &= \overline{x} - 0\\ \overline{x} &= 2\end{aligned}$$

04. first three moments about 2 : 1 , 16 , 40

Find mean ; variance and third central moment

SOLUTION :

A = 2 ; $\mu_1(a)$ = 1 ; $\mu_2(a)$ = 16 ; $\mu_3(a)$ = 40

$\mu_1 = 0$

$$\begin{aligned}\mu_2 &= \mu_2(a) - \mu_1(a)^2\\ &= 16 - (1)^2\\ &= 15\end{aligned}$$

VARIANCE = μ_2 = 15

$$\begin{aligned}\mu_3 &= \mu_3(a) - 3\mu_1(a)\,.\,\mu_2(a) + 2\mu_1(a)^3\\ &= 40 - 3(1)(16) + 2(1)^3\\ &= 40 - 48 + 2\\ &= -6\end{aligned}$$

$$\begin{aligned}\mu_1(a) &= \overline{x} - A\\ 1 &= \overline{x} - 2\\ \overline{x} &= 3\end{aligned}$$

MEAN = 3

05. first 3 moments about 7 calculated from a set of 9 observations are 0.2 ; 19.4 and −41 respectively . Find the mean , variance and the second raw moment of the distribution

SOLUTION :

$A = 7$; $\mu_{1(a)} = 0.2$; $\mu_{2(a)} = 19.4$; $\mu_{3(a)} = -41$

$\mu_1 = 0$

$\mu_2 = \mu_{2(a)} - \mu_{1(a)}^2$
$= 19.4 - (0.2)^2$
$= 19.4 - 0.04$
$= 19.36$

VARIANCE = $\mu_2 = 19.36$

$\mu_{1(a)} = \overline{x} - A$

$0.2 = \overline{x} - 7$

$\overline{x} = 7.2$

MEAN = 7.2

VARIANCE = 19.36

$\sigma 2 = 19.36$

$$\frac{\Sigma x^2}{n} - x^2 = 19.36$$

$$\frac{\Sigma x^2}{n} - (7.2)^2 = 19.36$$

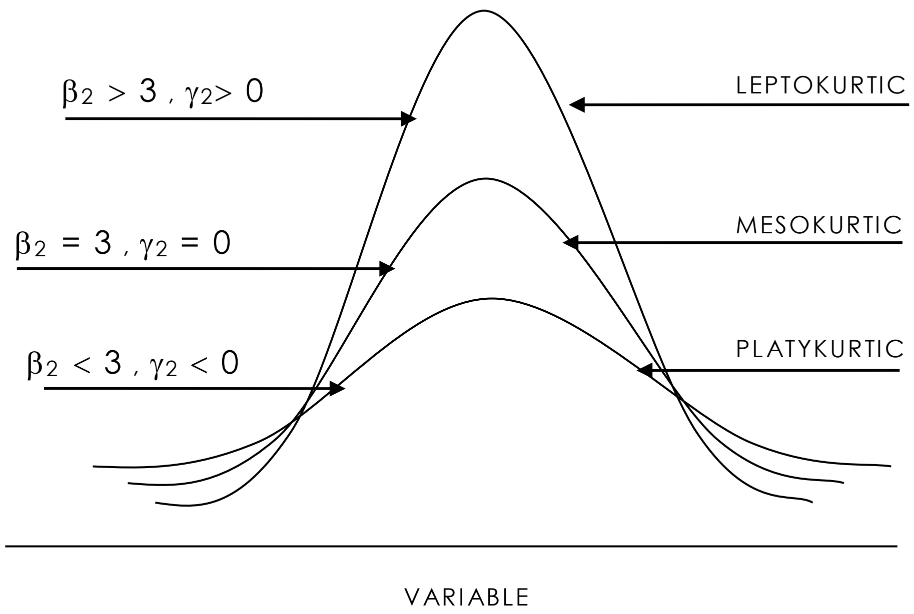
$$\frac{\Sigma x^2}{n} - 51.84 = 19.36$$

$$\frac{\Sigma x^2}{n} = 71.2$$

SECOND RAW MOMENT

$$\mu_2' = \frac{\Sigma x^2}{n} = 71.2$$

KURTOSIS



- The peakedness of a distribution is known as KURTOSIS .
- Pearsonian coefficients β_2 and γ_2 measures the kurtosis of the distribution
- $\beta_2 = \dfrac{\mu_4}{\mu_2 2}$ & $\gamma_2 = \beta_2 - 3$

Q1. the first four moments about 4 are 1 , 4 , 10 , 46 . Find Personian's coefficients of kurtosis

SOLUTION :

$A = 4$; $\mu_{1(a)} = 1$; $\mu_{2(a)} = 4$; $\mu_{3(a)} = 10$; $\mu_{4(a)} = 46$

$$\mu_2 = \mu_{2(a)} - \mu_{1(a)}^2$$
$$= 4 - (1)^2$$
$$= 3$$

$$\mu_3 = \mu_{3(a)} - 3\mu_{1(a)} . \mu_{2(a)} + 2\mu_{1(a)}^3$$

NOT REQUIRED

$$\mu_4 = \mu_{4(a)} - 4\mu_{1(a)} . \mu_{3(a)} + 6\mu_{2(a)} . \mu_{1(a)}^2 - 3\mu_{1(a)}^4$$
$$= 46 - 4(1)(10) + 6(4)(1)^2 - 3(1)^4$$
$$= 46 - 40 + 24 - 3$$
$$= 70 - 43$$
$$= 27$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{27}{(3)^2} = 3$$

$$\gamma_2 = \beta_2 - 3 = 3 - 3$$
$$= 0$$

COMMENT

DISTRIBUTION IS MESOKURTIC

Q2. the first four moments about 4 are –1 ,17 , –30 , 308 . Find Personian's coefficients of kurtosis

SOLUTION :

$A = 4$; $\mu_{1(a)} = -1$; $\mu_{2(a)} = 17$; $\mu_{3(a)} = -30$; $\mu_{4(a)} = 308$

$$\mu_2 = \mu_{2(a)} - \mu_{1(a)}^2$$
$$= 17 - (-1)^2$$
$$= 16$$

$$\mu_3 = \mu_{3(a)} - 3\mu_{1(a)} . \mu_{2(a)} + 2\mu_{1(a)}^3$$

NOT REQUIRED

$$\mu_4 = \mu_{4(a)} - 4\mu_{1(a)} . \mu_{3(a)} + 6\mu_{2(a)} . \mu_{1(a)}^2 - 3\mu_{1(a)}^4$$
$$= 308 - 4(-1)(-30) + 6(17)(-1)^2 - 3(-1)^4$$
$$= 308 - 120 + 102 - 3$$
$$= 410 - 123$$
$$= 287$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{287}{(16)^2} = \frac{287}{256} = 1.12$$

$$\gamma_2 = \beta_2 - 3 = 1.12 - 3$$
$$= -1.88$$

COMMENT

DISTRIBUTION IS PLATYKURTIC

Q3. the first four raw moments about origin are 2 , 20 , 40 , 800 . Find Personian's coefficients of kurtosis

SOLUTION :

$A = 0 ; \mu_1(a) = 2 ; \mu_2(a) = 20 ; \mu_3(a) = 40 ; \mu_4(a) = 800$

RAW MOMENTS

$\mu_2 = \mu_2(a) - \mu_1(a)^2$

$= 20 - (2)^2$

$= 20 - 4$

$= 16$

$\mu_3 = \mu_3(a) - 3\mu_1(a) . \mu_2(a) + 2\mu_1(a)^3$

NOT REQUIRED

$\mu_4 = \mu_4(a) - 4\mu_1(a) . \mu_3(a) + 6\mu_2(a) \cdot \mu_1(a)^2 - 3\mu_1(a)^4$

$= 800 - 4(2)(40) + 6(20)(2)^2 - 3(2)^4$

$= 800 - 320 + 480 - 48$

$= 1280 - 368$

$= 912$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{912}{(16)^2} = \frac{912}{256} = 3.56$$

$\gamma_2 = \beta_2 - 3 = 3.56 - 3$

$= 0.56 > 0$

COMMENT

DISTRIBUTION IS LEPTOKURTIC

Q4. if $\mu_2 = 16$ & $\mu_4 = 1024$. Find Personian coefficient of Kurtosis

SOLUTION

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{1024}{(16)^2} = \frac{1024}{256} = 4$$

$\gamma_2 = \beta_2 - 3 = 4 - 3$

$= 1 > 0$

COMMENT

DISTRIBUTION IS LEPTOKURTIC

Q5. if $\mu_2 = 4$ & $\gamma_2 = -0.4$. Find μ_4

SOLUTION

$\gamma_2 = \beta_2 - 3$

$-0.4 = \beta_2 - 3$

$\beta_2 = 2.6$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$2.6 = \frac{\mu_4}{(4)^2} \qquad \mu_4 = 41.6$$

Q6. if $\mu_4 = 108$ Find μ_2 if the distribution is MESOKURTIC

SOLUTION

Since the distribution is MESOKURTIC , $\beta_2 = 3$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$3 = \frac{108}{\mu_2^2}$$

$\mu_2^2 = 36 \qquad \therefore \mu_2 = 6$

Q7. If SD = 2 . Comment on KURTOSIS if μ_4

a) 50 b) 44 c) 48

SOLUTION

$\mu_2 = \sigma^2 = 4$

a) $\mu_2 = 4$; $\mu_4 = 50$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{50}{(4)^2} = 3.125$$

$\gamma_2 = \beta_2 - 3 = 3.125 - 3$

$= 0.125 > 0$; DISTRIBUTION IS LEPTOKURTIC

b) $\mu_2 = 4$; $\mu_4 = 44$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{44}{(4)^2} = 2.75$$

$\gamma_2 = \beta_2 - 3 = 2.75 - 3$

$= -0.25 < 0$; DISTRIBUTION IS PLATYKURTIC

c) $\mu_2 = 4$; $\mu_4 = 48$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{48}{(4)^2} = 3$$

$\gamma_2 = \beta_2 - 3 = 3 - 3$

$= 0$; DISTRIBUTION IS MESOKURTIC